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SEARCH FOR ENEMY TARGETS

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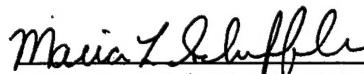
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**PREFACE**

The work presented in this technical report was a joint effort between researchers at the Air Force Research Laboratory (AFRL) and USA National Research Council. The problems of search and attack of enemy targets are research and development.

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# Search of Enemy Targets

(Statement of Problem, Discussions, and Some Solutions)

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## Abstract

The Persian Gulf (Desert Storm) and Afghanistan wars show that the USA Air Force needs Uninhabited Combat Aerial Vehicles (UCAVs) to search out, observe, recognize, and attack enemy targets. Presently, the Munitions Directorate of the Air Force Research Laboratory is placing emphasis on the guidance and control of autonomous munitions with characteristics similar to UCAVs, with active research being conducted in the areas of individual and cooperative control, search, and attack. These concepts involve the cooperative behavior of multiple munitions operating in a noisy and adversarial environment. It is envisioned that effective solutions of these problems will involve a combination of top-level operations research/artificial intelligence type of decision making combined with distributed control, distributed estimation, and real-time trajectory optimization. The authors formulate some problems from the noted field and consider methods of their solutions.

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## Introduction

There are numerous books and articles on the topics of searching for, observing of, and detection of targets [1]-[13]. In [1] Washburn presents a review of various search problems including localization, tracking, false alarms, false targets, optimal search, optimal planning, optimization of the detection threshold, etc. He also provides known techniques for their solution. In [2] editors Haley and Stone provide a collection of interesting articles about the above topics, which include applications in rescue, surveillance, exploration, medicine, industry, and clearance. In [3] Stone reviewed methods of optimal planning and searching for real and false targets. In [4] Broun studied optimal search for a target moving in discrete time and space. Steward [5] studied a constrained searcher motion. Kisi [6] researched optimal stopping of search. In [7] Enns et al addressed the problem of cooperative search in a horizontal plane subject to atmospheric and other disturbances and minimized the total path length traveled by the group of vehicles. In [8] Chandler et al investigated the complexity in cooperative control, implementing a hierarchical decomposition where team vehicles are allocated to subteams using set partition theory. In [9] Earl and D'Andrea used an optimization approach for synthesizing control strategies for cooperative multi-agent behavior. Finally, R.A. Murphrey [10]-[12] developed algorithms for collective and cooperative systems.

In this paper we investigate some specific search, observation, and attack problems.

The US Air Force is planning to use uninhabited combat aerial vehicles (UCAVs) extensively for searching, and attacking enemy targets. These concepts involve the cooperative behavior of multiple UCAVs operating in and adversarial environment. The effective solutions of these problems involve a combination of top-level operations research/artificial intelligence type of

decision making combined with distributed control, distributed estimation, and real-time trajectory optimization. It is doubtful that each vehicle will have global information. Rather, each vehicle will receive limited noisy information concerning the states of the other vehicles, and possibly only information concerning the states of its nearest neighbors.

At MN, researchers are persevering development of autonomous munitions with attributes similar to those of USAV. A combination of UCAVs and autonomous munitions will revolutionize air-to-surface combat, providing faster reaction to time critical targets in heavily defended regions. Since the basic research for munitions and UCAV are so similar, we will use the terms interchangeably in the remainder of the paper.

Some of the specific items relating to group vehicle behavior which we will investigate in this paper are:

- search strategies for air vehicles in pursuit of fixed and mobile targets
- strategies for searching important areas within a region and for continual observation
- techniques for calculating the number of required air vehicles for various tasks
- methods for selecting the best vehicle from a group for the purpose of attacking a target

The report is organized as follows:

In section 1, the planar equations of motion of a flight vehicle are presented.

In section 2, a general description of the problems of the search and detection of enemy targets is presented.

In section 3 and 4, search strategies are developed for locating fixed and mobile targets, respectively, using a single flight vehicle.

In section 5, the problem of continual observation is studied.

In section 6, search strategies for multiple vehicles are presented.

In section 7, selection of an appropriate attack vehicle from a group of attack vehicles is addressed.

In section 8, general rules of behavior of the attack vehicle are developed for the annihilation of the target.

The paper is then closed with a Summary section.

**History.** The US Air Force is studying the possibility use of small munitions and small uninhabited aerial vehicles for searching, and attacking enemy targets. These concepts involve the cooperative behavior of multiple UAVs operating in a noisy environment. The effective solutions of these problems involve a combination of top-level operations research/artificial intelligence type of decision making combined with distributed control, distributed estimation, and real-time trajectory optimization. It is doubtful that each vehicle will have global information. Rather, each vehicle will receive limited noisy information concerning the states of the other vehicles, and possibly only information concerning the states of its nearest neighbors.

Some of the specific problems that may arise in this area are:

- (1) minimum fuel consumption enroute to a search area or target,
- (2) optimal multiple vehicle search which might consist of the most efficient (minimum fuel) or most effective (fastest) way,
- (3) cooperative search a given area,
- (4) cooperative real-time path planning,
- (5) trajectory optimization when targets of opportunity are found,
- (6) cooperative multiple vehicle rendezvous in preparation for a cooperative attack,
- (7) constrained optimization and constrained feedback control to avoid vehicle collisions,  
and

(8) optimal distribution for a given number of search aircraft.

Integrating a design process by simultaneously considering the requirements of different design aspects is necessary to achieve optimum performance and group (cooperative) behaviors of future uninhabited, highly intelligent aircraft and aerospace vehicles. The traditional approach of designing a vehicle has been to sequentially satisfy the requirements of each discipline – material selection, structural geometry, control system or flight regimes. There are many fundamental issues that require investigation for a better understanding of the integration process and its consequences. The basic objective of multidisciplinary design and behavior is to integrate the various disciplines that constitute the environment of a flight vehicle. The goal of modern design and group control is to optimize the total system rather than individual components, permitting the conflicting requirements of the subsystems to be handled much more effectively in getting optimal solutions. Collection of design and group behavior is a huge optimization problem consisting of libraries of variables, constraints, performance functions, and group artificial intelligence. As the system becomes more and more complex, a creative designer needs to supplement intuition with computational tools in order to verify the validity of new concepts.

There are two areas of potential optimization for any combination of unmanned aircraft: design of optimal group behavior, and optimal flight. One approach deals with geometric optimization. The other approach deals with the optimal real-time cooperative planning and trajectory optimization. Achieving the real-time or quasi-real-time trajectory optimization will enhance additional aircraft operating efficiencies.

Below we discuss some of above noted problems and give some simple solutions.

### Current and Planned Status of UAVs

Afghanistan is presenting the U.S. with a new kind of war. Among the fundamental differences is the fact there are only bases for warplanes – support aircraft – in the surrounding countries.

The resulting “tyranny of distance” is a twist that makes the conflict in Afghanistan far different from those in the Balkans or the Persian gulf, according to Lt. Gen. Thomas Keck, chief of the 8<sup>th</sup> Air Force. His command encompasses both strategic bombers and information warfare, and it relies heavily on reconnaissance.

Currently the AF USA uses the Global Hawk for searching and recognition uses of unmanned reconnaissance aircraft. The Global Hawk has a 35-hr. endurance and 100-mi. surveillance range. Another search system is the E-8 Joint-STARS. It's ground surveillance radar which can look more than 200 mi. into enemy territory.

The Global Hawks – along with the hellfire-missile-equipped Predator UAV, F-15E, and RC-135 Rivet Joint signals intelligence-endurance platforms – will be major parts of an extremely fast “sensor-to-decision maker” system. It will let allied forces spot, identify and strike moving targets within a few minutes. These aircraft, many of them with synthetic aperture radar and GPS-guided bombs, will be able to see and strike through the heavy clouds produced by Afghanistan's fierce, high desert winter weather.

Global Hawks participating in the Afghan conflict are expected to carry a sensor suite that includes electro-optical and infrared cameras, synthetic aperture radar, and the LR-100 electronic intelligence-gathering system. This latter system was installed in the Global Hawks for Australia deployment and was never removed.

Deployment of the Global Hawk and joint-STARS will give commanders a continuous, wide-area surveillance of significant portions of territory that they have lacked until now. For example, Predator UAVs give commanders a good view of a target, but only if they know where to look on the battlefield. The Global hawk and Joint-STARS will provide wide-angle observations that can cue both manned and unmanned strike aircraft.

In Pentagon planning, the assembled forces conduct distinct missions in the "find, fix, target, track and engage" process of modern combat. Finding targets is assigned to Rivet Joint, infrared-sensor-equipped satellites and the EA-6B Prowler. A target's position is then fixed by the U-2 or Predator UAV. Target data is provided by the Air Operations Center, E-3 AWACS, EC-130 Airborne Command and Control Center and joint-STARS. If the target needs to be tracked for a period instead of being struck immediately, the mission goes to Joint-STARS or Global Hawk. Finally, the target is engaged. It can be disrupted by information attack – possibly a computer assault or jamming by the EA-6B or EC-130 Compass Call. - or, targets can be destroyed by antiradar missiles, cruise missiles, Hellfire, EGBU-15 or other precision, air-launched weapons. Global Hawk and the smaller, medium-altitude RQ-1A Predator UAV are expected to be key elements in a fast reaction strike system designed to hit targets within 5 min. of detection.

General Atomics has been developing the Predator-B on its own, but has sold three of the aircraft to the U.S. Air Force. The turboprop-powered Predator-B is able to fly higher than Predator, carry more payload, and go faster. The company has also developed a turbojet version, but the Air Force isn't interested because it has much less endurance. Predator-B's mission characteristics approach those of Global Hawk at much lower cost. Predator-B flies at 45,000 ft, and can carry about 700 lb. internally and 1,000 lb. Externally, rather than the advertised 1,500

lb. One option under consideration by the Service is flying Predator-B into a target area at 45,000ft. and then having it drop down to a lower altitude.

The Defense Advanced Research Projects Agency (DARPA) and Frontier Systems logged the "first flight" of their unusual A160 hummingbird unmanned helicopter that is built to fly much longer and farther than conventional rotorcraft. Hummingbird is supposed to be able to stay aloft for more than 40 hr. with a 300-lb. Payload. Maximum altitude is estimated to be 30,000ft.

The U.S. Army has initiated development of a signals intelligence (sigint) capability for its Shadow-200 tactical unmanned aerial vehicle, with the hope of addressing a long-standing requirement that has repeatedly proven difficult to meet. The Shadow-200 has 60 lb. of payload, with about 1 cu. ft. of volume and 500 watts of available power. When operating the sigint payload, the UAV would have to be stripped of its electro-optical/infrared sensor.

#### **Performance of RQ-4A Global Hawk:**

Loiter speed - 635 km/h; 395mph.

Loiter altitude 15,240 – 19,810 m (50,000 – 65,000 ft).

Service ceiling - 19,810m (65,000 ft).

Max range - 12,000 n miles (22,224 km; 13,809 miles).

On-station endurance at 3,000 n miles - 24 h.

The Boeing and DARPA developed X-50 Dragonfly unmanned vehicle. An innovative canard rotor/wing design combines the helicopter's hover capabilities with the high-subsonic speed of a fixed-wing aircraft. The 1,400-lb experimental aircraft is 18 ft long with a rotor diameter of 12 ft, appropriately sized to meet DARPA's requirement to demonstrate the concept's suitability as a VTOL UAV.

## 1. Equations of Flight Vehicle Motion.

Below there are common aircraft equations. Modifications of it will be used for consideration of particular cases of UAVs maneuvers.

Assume that the Earth is planar, non-rotating, and the thrust is directed along the vehicle's velocity vector. In the conventional non-rotating, but translating, coordinate system, connected to the center gravity of the thrusting, lifting vehicle, the basic equations of motion are:

a) Kinematics equations:

$$dH/dt = V \sin \gamma, \quad (1)$$

$$dx/dt = V \cos \gamma \cos \psi, \quad (2)$$

$$dy/dt = V \cos \gamma \sin \psi, \quad (3)$$

$$ds/dt = V \cos \gamma, \quad (4)$$

b) Dynamic equations:

$$dV/dt = [V_e \beta - D(\alpha, V, H)]/m - g \sin \gamma. \quad (5)$$

$$V d\gamma/dt = L(\alpha, V, H) \cos \sigma / m \cos \gamma - g \cos \gamma, \quad (6)$$

$$V d\psi/dt = L(\alpha, V, H) \sin \sigma / m \cos \gamma. \quad (7)$$

$$dm/dt = -\beta. \quad (8)$$

where:

$H$  - altitude;

$t$  - time;

$V$  - vehicle speed;

$x$  - coordinate along axis x;

$y$  - coordinate along axis y;

$s$  - way along trajectory;

$V_e(H, M)$  - coefficient of thrust;

$D$  - drag;

$L$  - lift force;

$g = 9.81 \text{ m/sec}^2$  coefficient of gravity;

$\beta$  - fuel consumption;

$\gamma$  - the flight path angle measured positive upward from the local horizontal plane (see fig.1);

$\psi$  - the heading angle measured positive to the left of the initial trajectory;

$\sigma$  - the bank angle;

$m$  - mass of vehicle;

$\xi$  - sideslip angle;

$T = V_e \beta$  - thrust;

$R_1$  - turning radius in vertical plane;

$R_2$  - turning radius in horizontal plane.

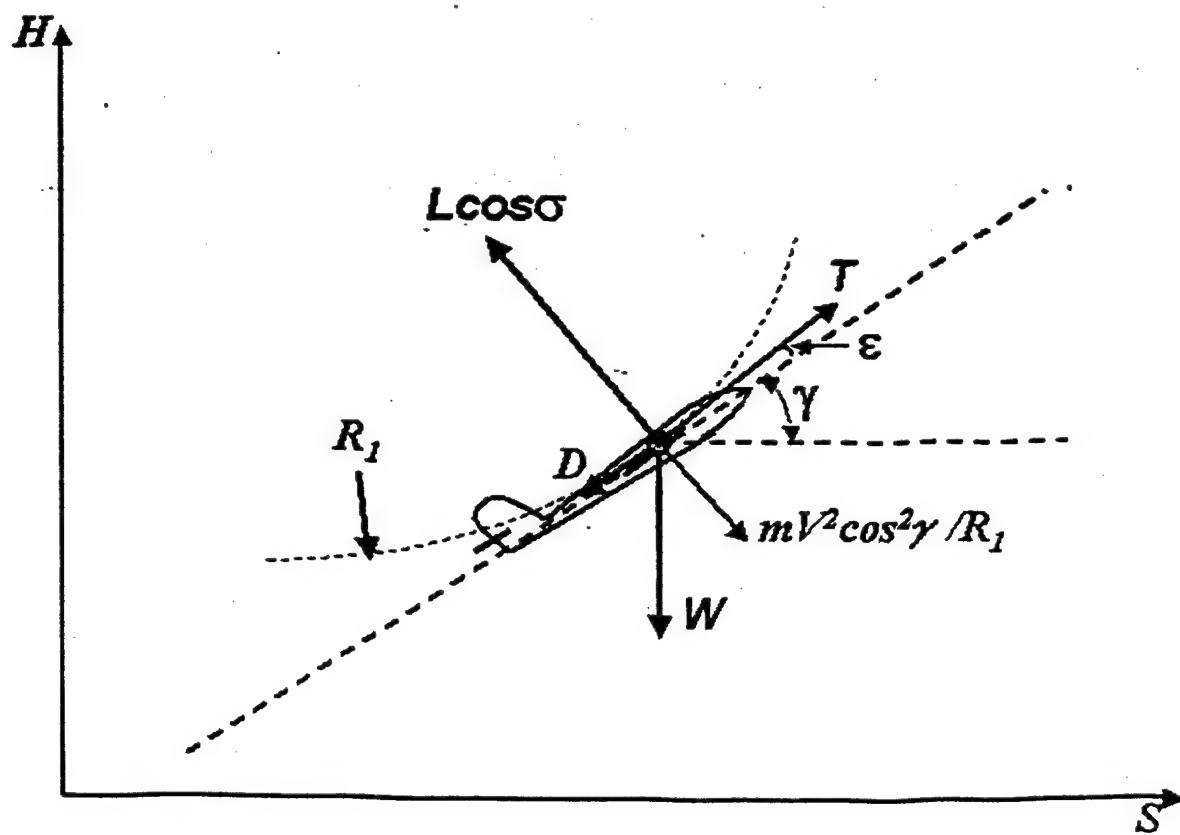


Fig.1. Forces acting on an aircraft in a vertical plane.

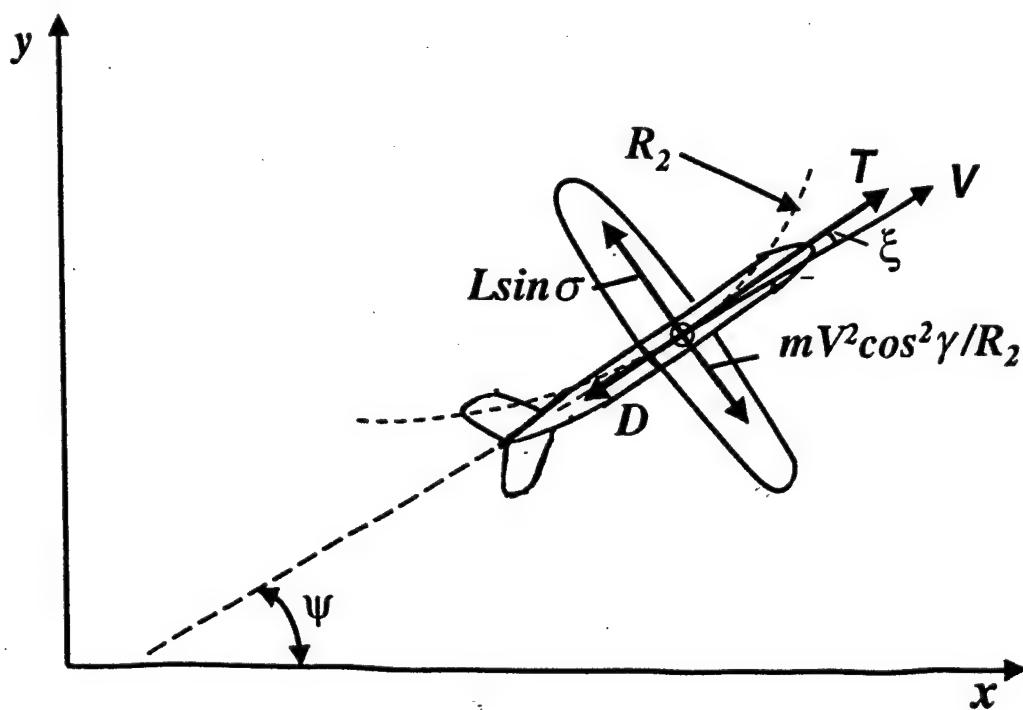


Fig.2. Forces acting on an aircraft in a horizontal plane.

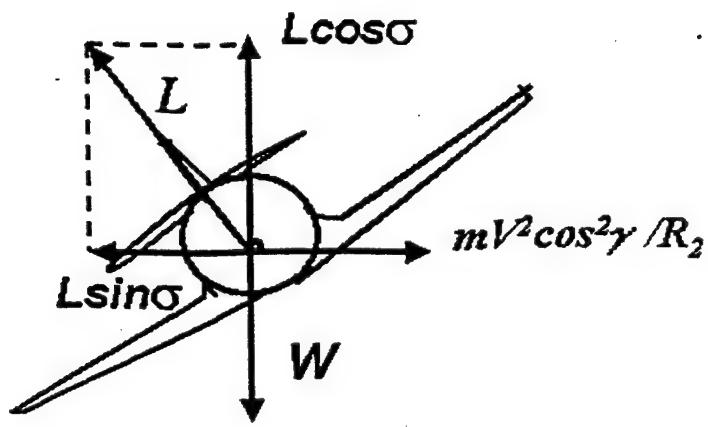


Fig.3. Forces acting on aircraft (forward view)

The angles used in equations (1)-(8) are represented in Figs. 1-3.

The controls are

$$\alpha, \sigma, \beta. \quad (9)$$

There are constraints:

$$L = C_L \rho V^2 S / 2, \quad (10)$$

$$D = C_D \rho V^2 S / 2, \quad (11)$$

$$C_L = C_{L\alpha} \alpha, \quad (12)$$

$$C_D = C_{D0} + K C_{L\alpha}^2 \alpha^2; \quad (13)$$

where :

$\alpha$  - angle of attack;

$C_{L\alpha}$ ,  $C_{D0}$ ,  $K$  - coefficients are functions of the Mach number  $M$ .

The angle-of-attack is constrained by

$$\alpha_{min} < \alpha < \alpha_{max} \quad (14)$$

The fuel consumption is constrained by

$$\beta_{min} < \beta < \beta_{max}. \quad (15)$$

The vehicle can also be limited by the overload  $n$

$$n < n_{max} \quad , \quad (16)$$

where

$$n = L/W ;$$

$W = mg$  is weight of vehicle.

Equations (1)-(16) must be satisfied when finding optimal trajectories.

## 2. General Description of the Problem of the Search and Determination of Enemy Targets

In this section the method of operations assumed for finding and attacking targets is explored.

The approach searching for enemy targets is shown on Figs. 4-5.

The method in Figure 4 features an unmanned aircraft surveillance located at a high altitude.

This vehicle makes a general search, recognizing targets, and communicates with a command center as well as lower altitude attack vehicles. The attack vehicles, notionally small UAV's or autonomous munitions, receive commands, data, and coordinates of the target from the search vehicle or the command center. The attack vehicles may also contribute to the search and conduct target recognition before attacking the target. Figure 5 illustrates the attributes of a possible nude area search weapon; the Lowest Autonomous Attack System (LOCAAS).

The front and top views of the area of observation are shown in Figures 6 and 7 respectively.

The form of the area depends on the type of scanning being conducted. If the scanning ray has a circular motion, the scanning area has the form depicted in Fig.7a. If the scanning ray has an oscillatory motion, the scanning area has the form depicted in Fig.7b. For the particular attack UCAV considered, the scanning area has the form depicted in Fig.7c since it is assumed that the UCAV can only attack targets located in its forward hemisphere.

**Fig.4. Cooperative search and attack (see next page).**

**Fig.5. The typical attack vehicle and its attack area (see next page).**

## Cooperative Attack

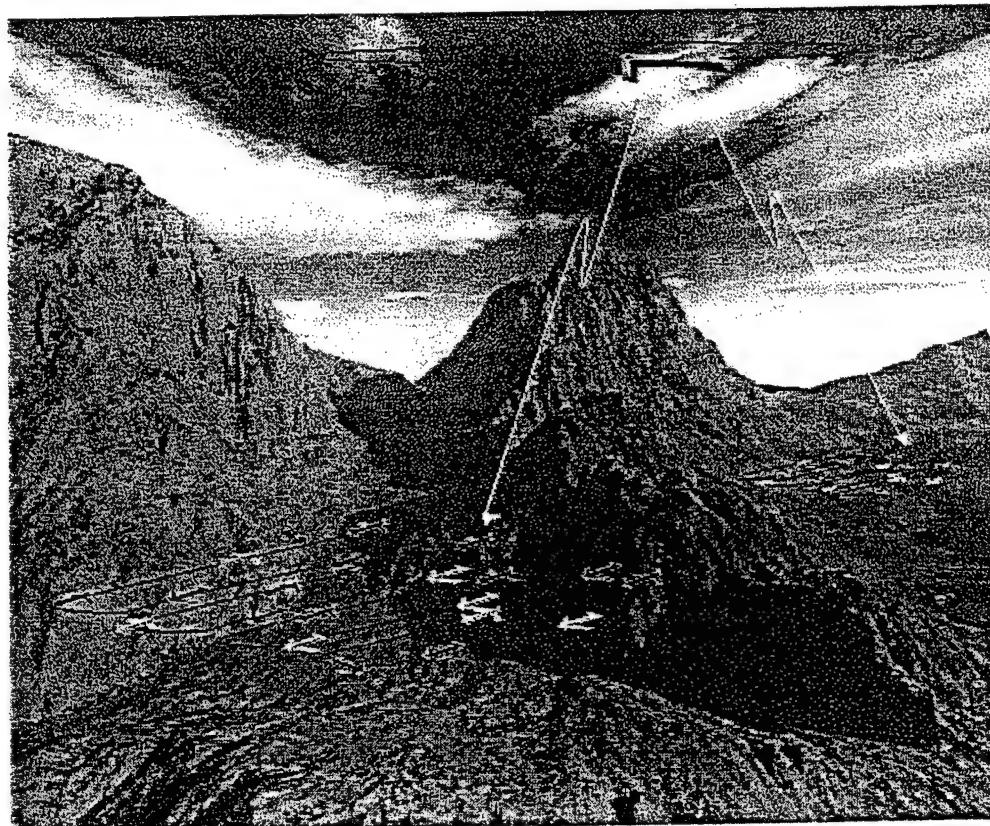
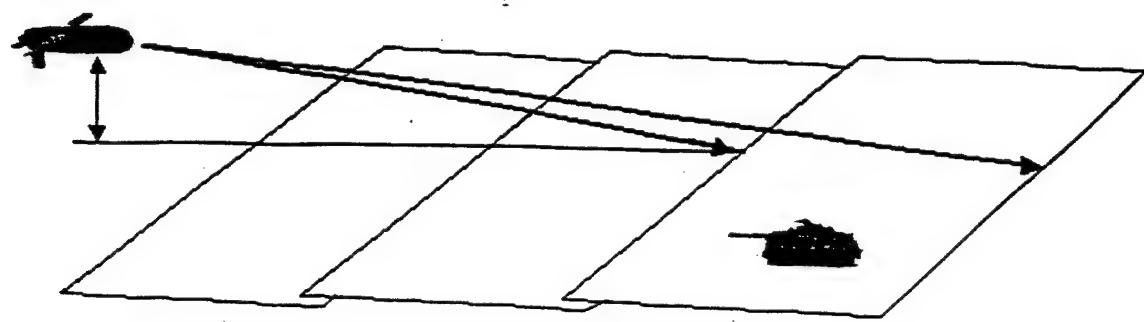
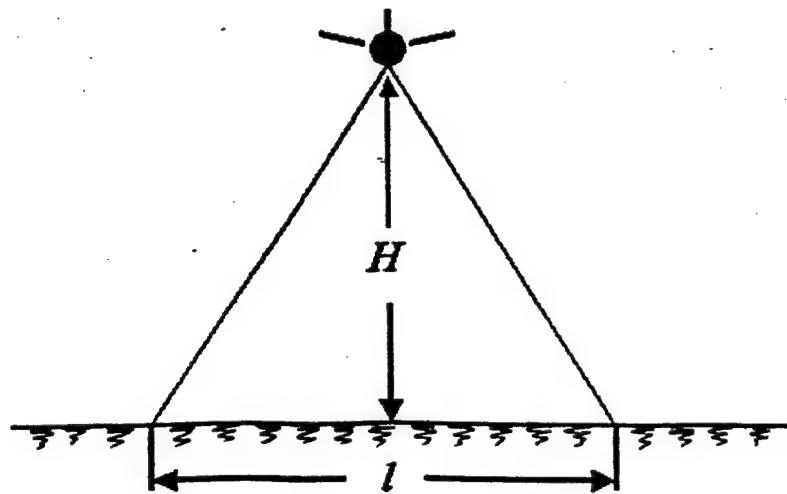


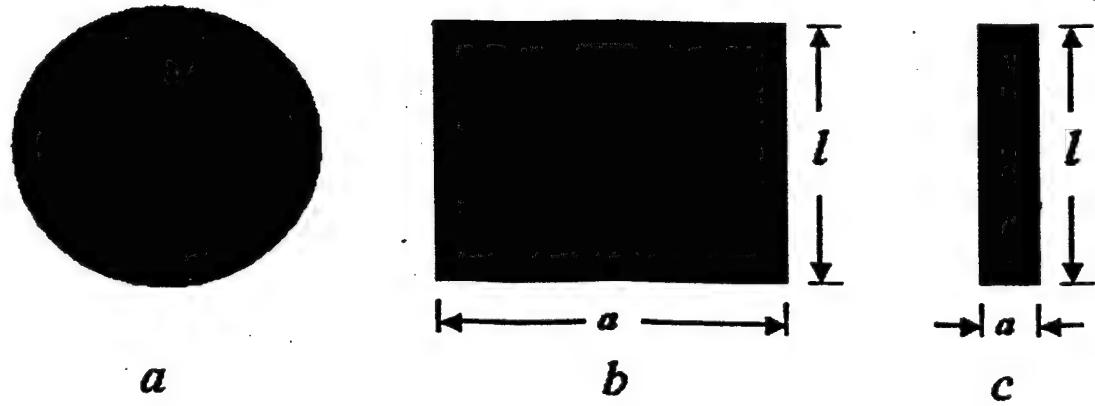
Fig.4



**Fig.5**



**Fig.6.** The typical observation angle of an observed aircraft (forward view).



**Fig.7a,b,c.** Typical observed areas (top view).

The width of the scanning beam  $l$  (Fig.6) is very important. From the scanning angle  $\omega$  and the height  $H$  (Fig.6), it can be calculated by

$$l = 2H \tan (\omega/2) . \quad (17)$$

The larger  $l$  is, the greater the area searched per second will be.

Using the kinematics equations (1)-(4) we will consider various trajectories in some typical situations.

### 3. Search strategy of a single flight vehicle and an fixed target

#### *The parallel sequence strategy of an observation*

Let us consider the following search strategy, which we denote as the parallel sequence strategy. In this strategy the vehicle has parallel trajectories (Fig.8).

$$d = 0.5l$$

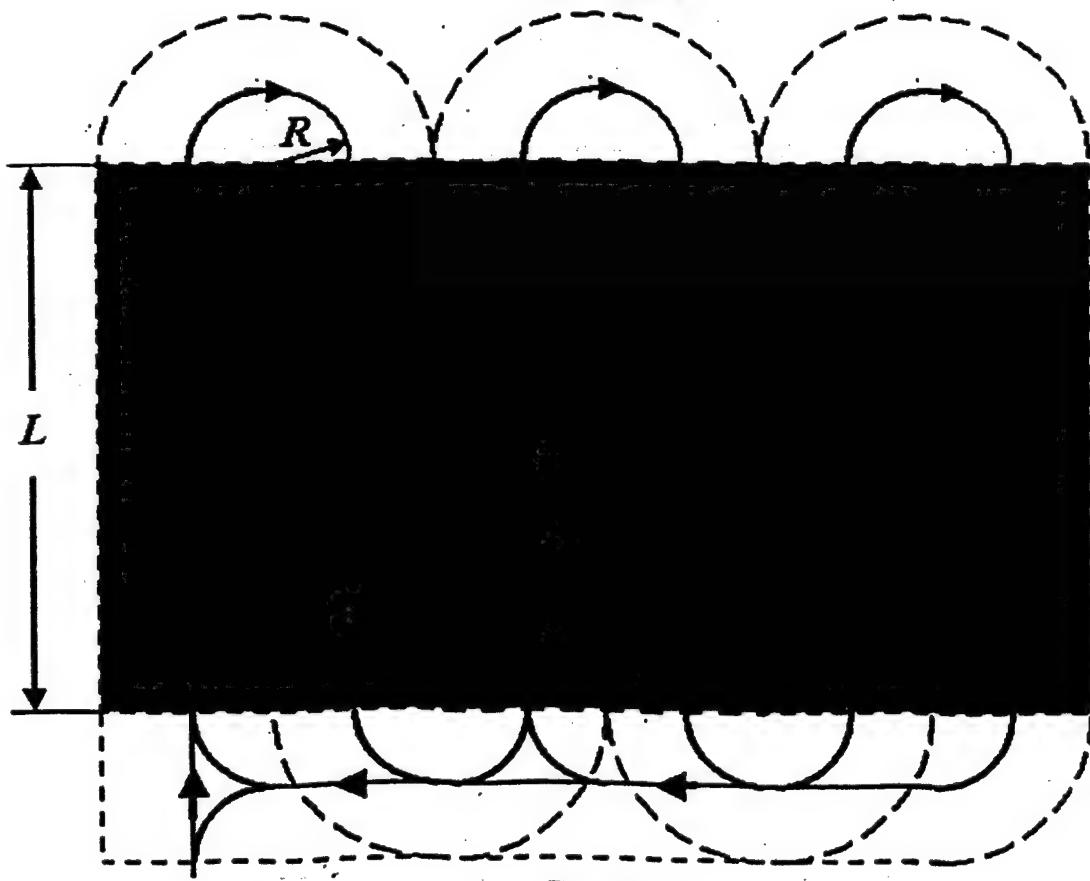


Fig.8. The parallel sequence strategy.

The vehicle observes the given area which has width  $L$  ( $L \gg l$ ;  $L \gg R$ ) and repeats this process with period  $P$ . The turning radius must be less than the observation width  $l$ . In this case we can ignore the additional areas at the end of the straight parallel trajectories. The parallel trajectories are separated a distance of  $2l$  and the observation area is fully observed. If the aircraft returns via path  $I$ , the observation period  $P$  is constant for any point in the area (Fig.9).

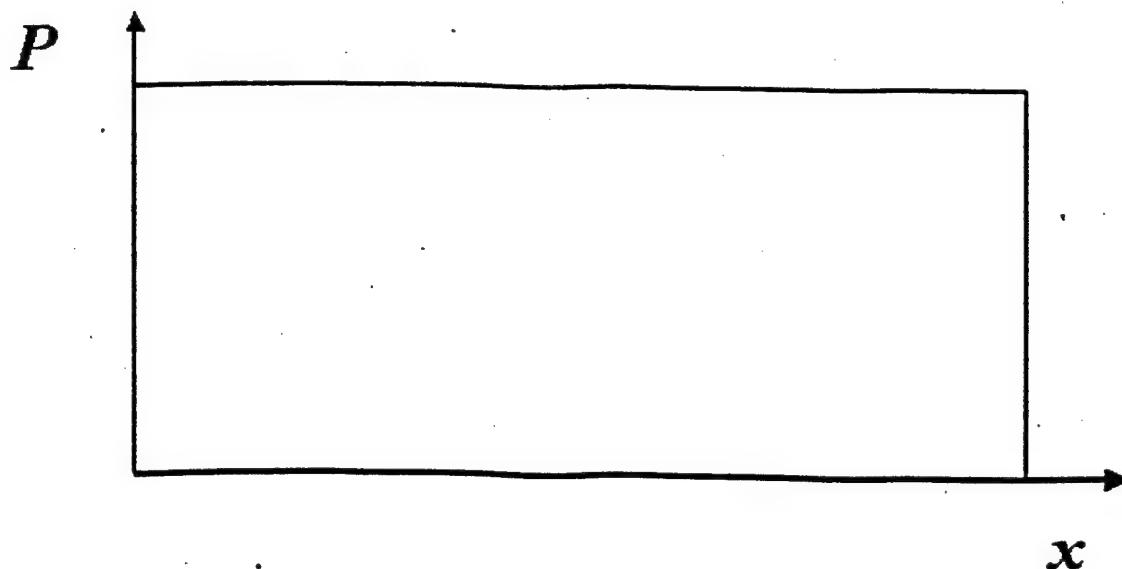


Fig.9. Constant observation period along the observed trajectory.

This period can be calculated by the equation

$$P = S/V, \quad (18)$$

where  $S$  is pathlength of one cycle.

The observation vehicle can return via the path shown in Fig.10. In this case, the observation period will vary for different points along the trajectory (Fig.11), depending on the location point and the direction of the vehicle's flight.

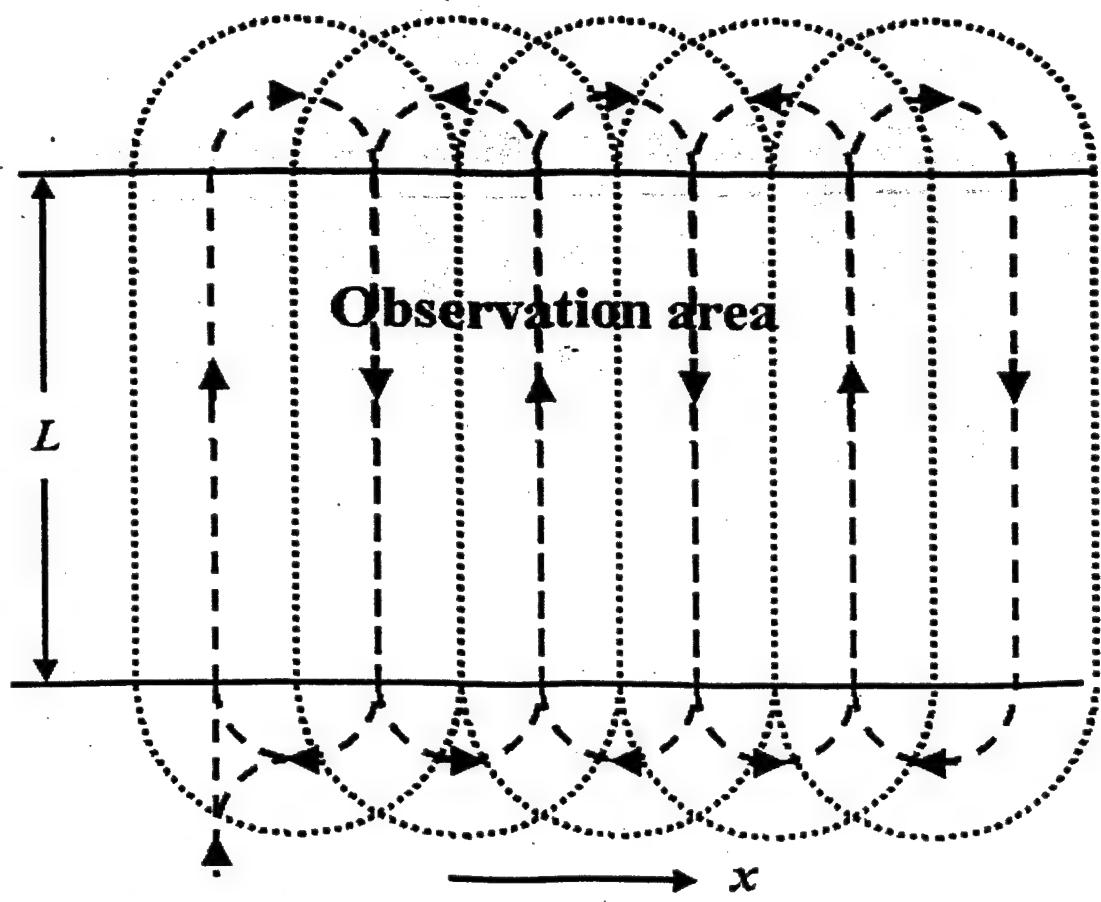


Fig.10. The oscillation method of searching.

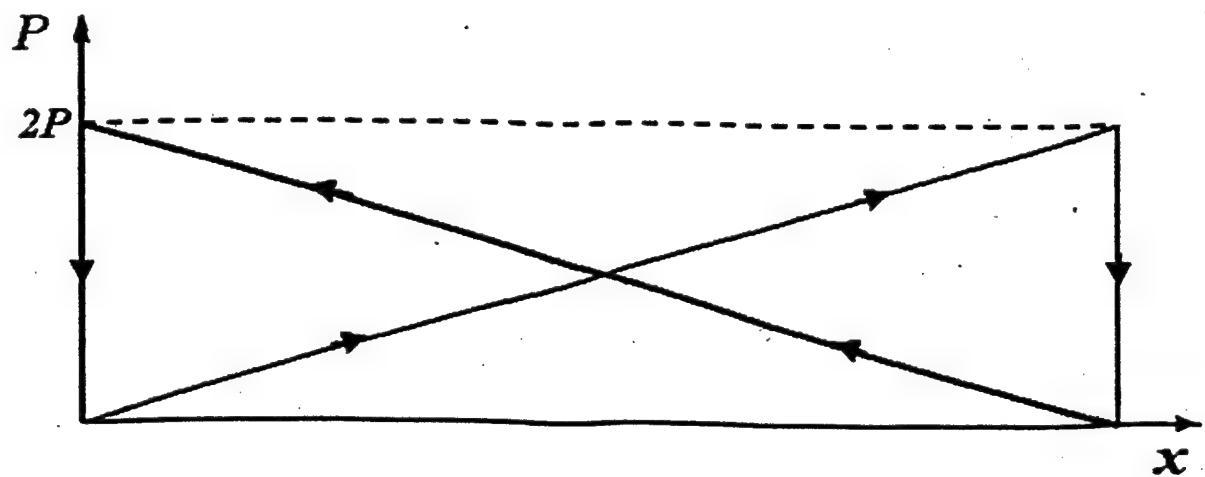


Fig.11. Change in the observed period for the oscillation trajectory.

### *The spiral strategy of observation*

The other method of observation is shown in Fig.12. The aircraft moves in a spiral trajectory. The distance is the same between branches of the trajectories and is equal to the observation width  $l$ .

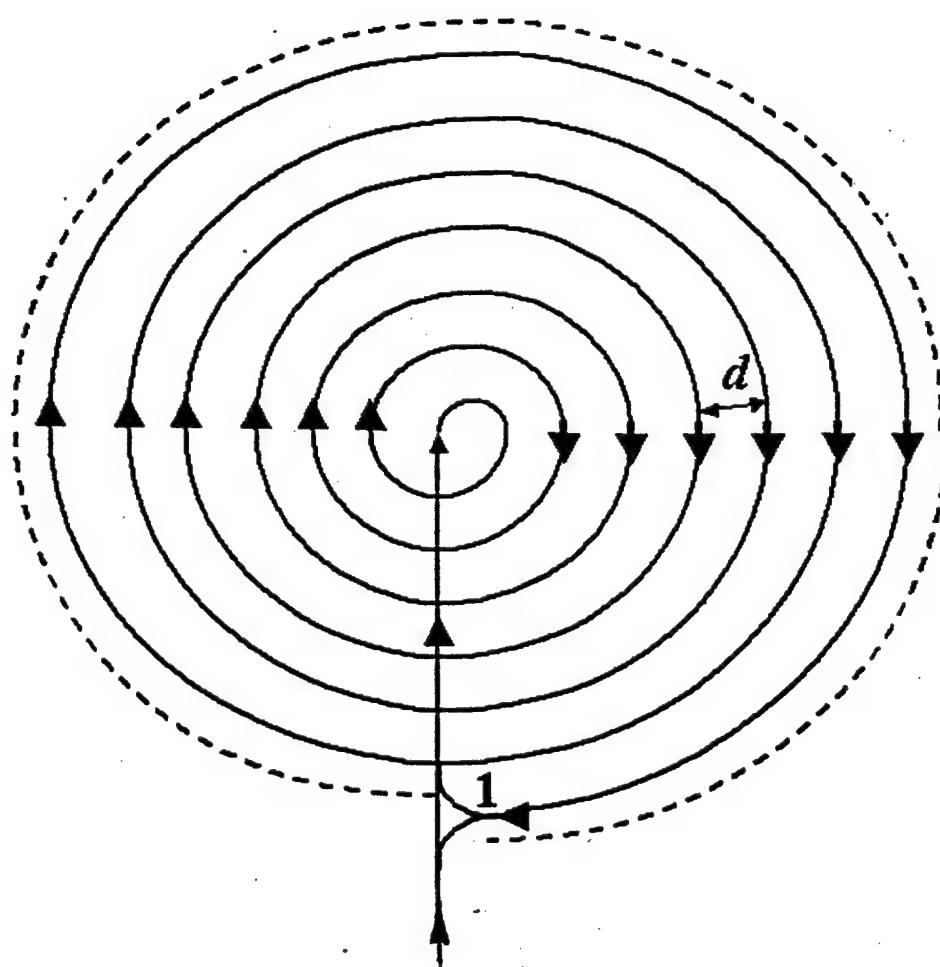


Fig.12. The spiral strategy of searching.

The equation of the spiral motion is:

$$\rho = a\varphi, \quad (19)$$

where

$\rho$  - radius of a spiral;

$\varphi$  - angle of a radius-vector;

$a$  - parameter,  $a=l/2\pi$ .

The length  $S$  of the spiral is

$$S = a\{\varphi(\varphi^2 + 1)^{0.5} + \ln[\varphi + (\varphi^2 + 1)^{0.5}]\}/2. \quad (20)$$

If the observation aircraft repeats this cycle via path 1 (Fig.13), the period  $P$  of the observation is constant at any point within the observation area and equals

$$P = S/V. \quad (21)$$

This method is appropriate when the observation area is circular.

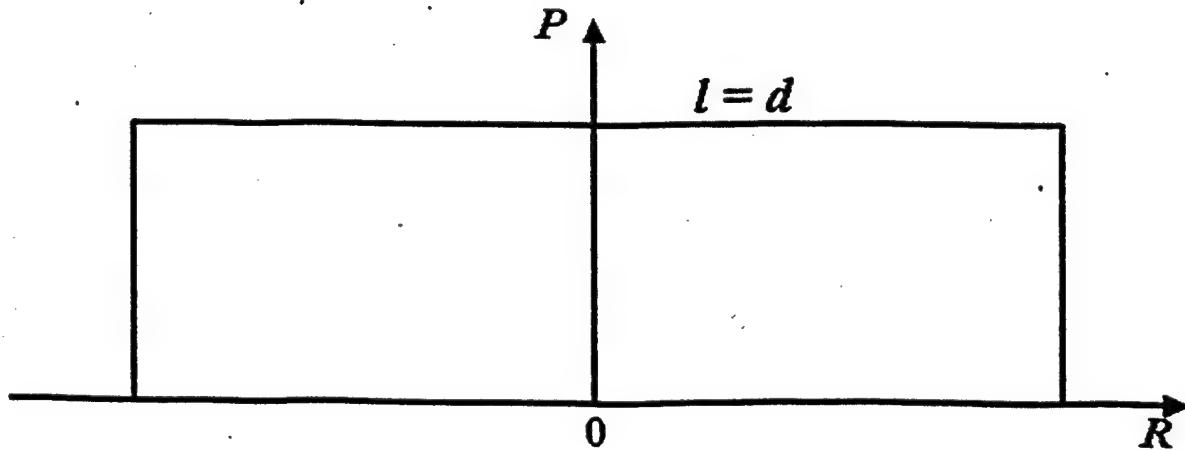


Fig.13. The period of observation for the spiral strategy.

If there is a higher probability that the target is located in the center of the spiral, it is better to take  $d < l$  and to decrease the period of observation in the center of the spiral. For example, if  $d = 0.5l$ , the period is decreased twofold in center of the spiral, but at the expense of being increased twofold in the peripheral region of the spiral.

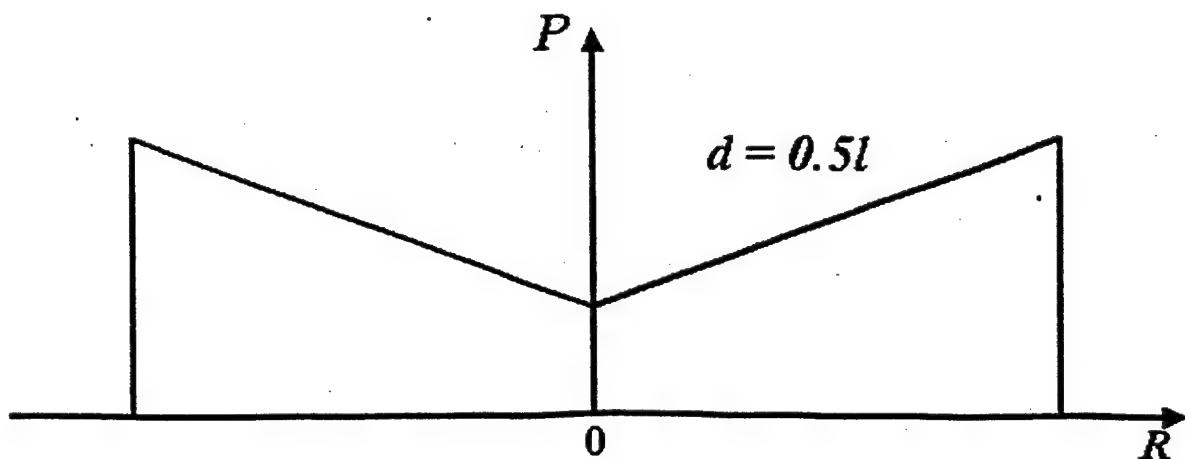


Fig.13a. The period of observation for the spiral strategy for the case  $d = 0.5l$ .

## 4. Searching for a Mobile Target

The problem becomes more complex when the target is mobile. The search depends on the ratio of the target velocity  $v$  to the search vehicle's velocity  $V$ . The search vehicle can lose the target if it uses the described above strategies, because in the period between the observations, the mobile target can move into a region which was previously observed. In this case, the result depends on the ratio  $v/V$ . If this ratio is small, the observation vehicle has total area coverage. If this ratio has some critical value, full observation is possible only in a limited area. And if the target has sufficient speed to pass through an observation area faster than the observation period, then the search should be performed randomly. For the two search strategies presented below, we assume that the ratio  $v/V$  is small ( $v/V \ll 1$ ).

### Parallel sequence search strategy

Consider the case where a mobile target is initially located at a lower position and the search aircraft is initially located at a top position. The searching aircraft moves with speed  $V$  and the target moves with speed  $v$  in a direction perpendicular to flight line (Fig. 14). When the searching aircraft moves from point  $A$  to point  $B$ , the target moves from the point  $C$  to point  $D$ .

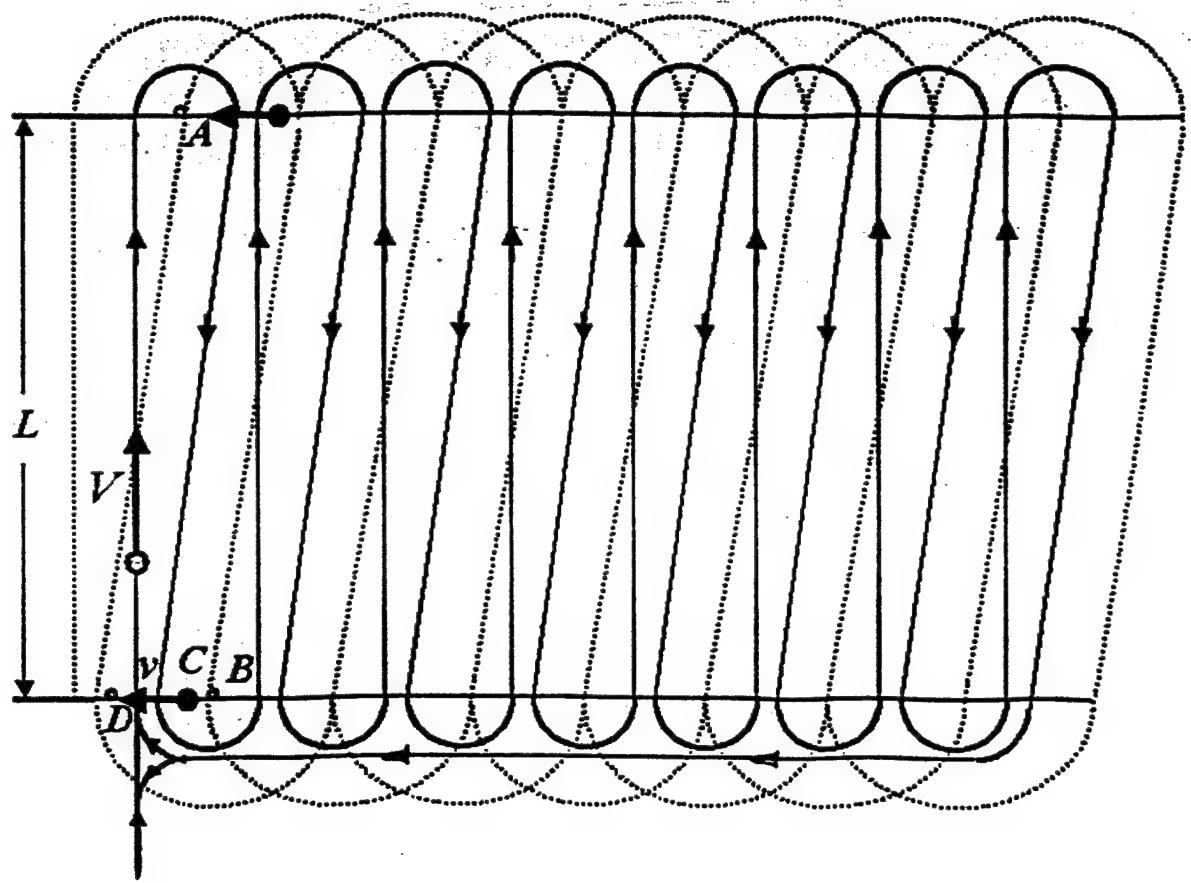


Fig.14. Parallel strategy for a mobile target.

This motion takes the time:

a) For the searching aircraft

$$T = L/V .$$

b) For the target

$$t = l/v .$$

For the full cover of the searching area, the time of the target must be less than the time of the searching aircraft

This means that

$$L/l < V/v. \quad (22)$$

If  $v=0$ , then the period of observation is

$$P_m = P_o.$$

If  $L/l = 2 V/v$ , then the period of the observation doubles:

$$P_m = 2 P_o,$$

or the observation area is cut in half:

$$A_m = 0.5 A_o.$$

If  $L/l = V/v$ ,

$$P_m = 0.$$

If

$$V/v > L/l,$$

it is impossible to effectively observe the given area for mobile targets.

### Spiral search strategy

The same problems above are present in the spiral search strategy. Consider the critical case when the target has the speed  $v$  and moves along a radial (Fig.15).

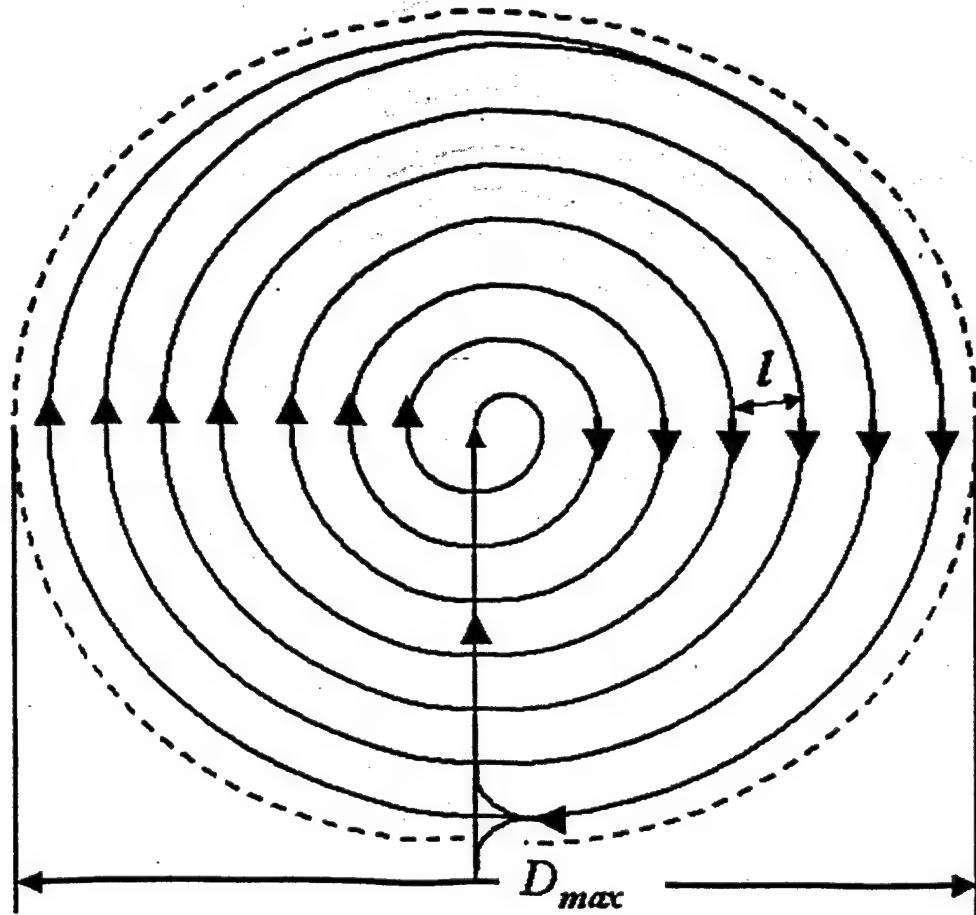


Fig.15. The spiral strategy for the mobile target.

The time of target movement is

$$t = L/v . \quad (23)$$

The time of one revolution of the search vehicle is

$$T = \pi D/V . \quad (24)$$

The time of the target movement must be less than the time of search

$$t < T . \quad (25)$$

This means that the maximum diameter of the full observation is

$$D_{max} = \pi V/v . \quad (26)$$

## 5. Continual Observation

If the target is very important, it needs to be constantly observed. This means that the observation period must be zero. Consider the case for a single observation aircraft.

Assume that the aircraft has the observation radius  $R$  (Fig. 7a) and minimum turning radius  $r$ . Suppose that the radius  $r$  is less the radius  $R$

$$r < R \quad (27)$$

and the searching aircraft flies in a circle of radius  $r$  (Fig.16). Then the region of the continual observation has radius (Fig.16)

$$R_{obs} = R - r \quad (28)$$

The same result will be true for the rectangular observation area (Fig.7b). It is impossible to create a continual observation area for the scanning area Fig.7c when  $a \ll l$ .

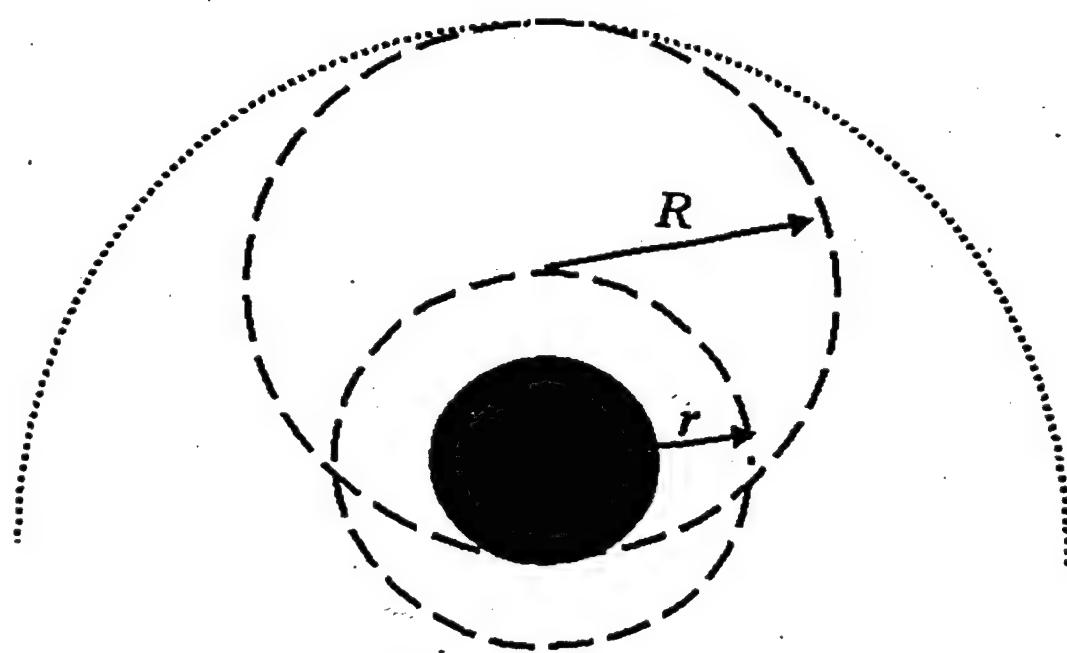


Fig.16a. Continual observation area.

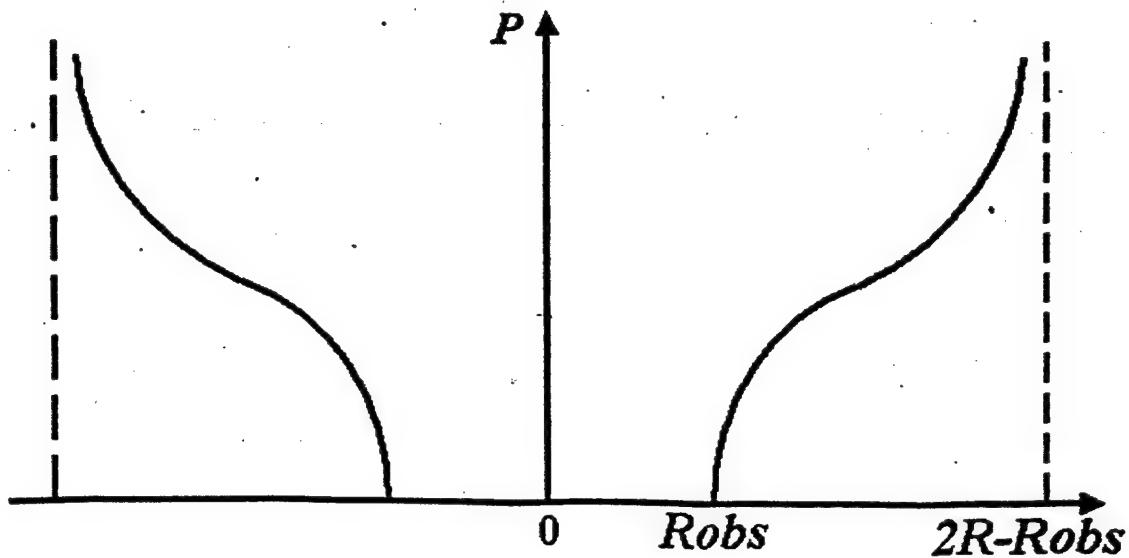


Fig.16b. Period of the observation for Fog.16a.

## 6. Searching Strategy for Multiple Vehicles

If there are multiple search aircraft, many methods for searching can be used. We can use the previous results in three ways.

1. The  $n$  searching aircraft can fly parallel routes along the desired observation direction which perpendicular to a frontal formation of aircraft (Fig.17). In this case, the aircraft increase the width of the observation line  $l$ . The frontal formation increases the searching capability by a factor of  $n$  for an immobile target and improves the search for a mobile target.

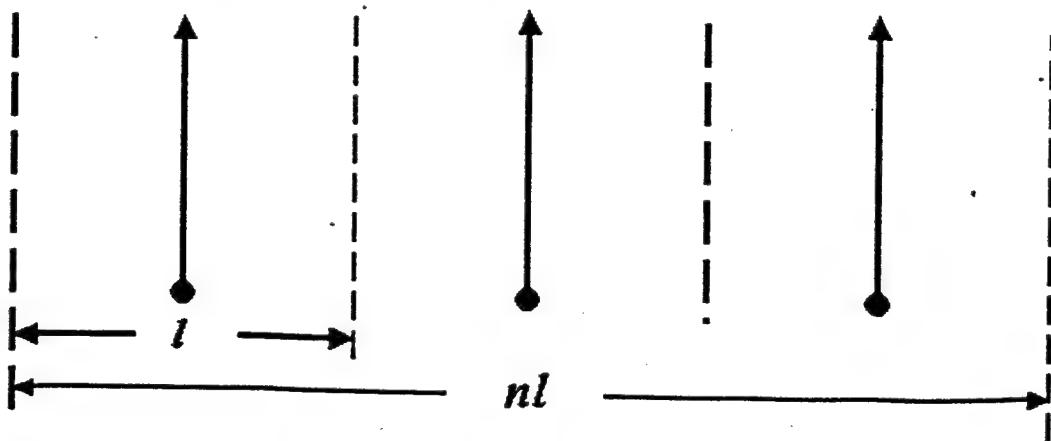


Fig.17. A frontal formation of a group of search aircraft.

2. The  $n$  searching aircraft can fly the same flight path in an equally-spaced one after another (Fig. 17-18) staggered formation.

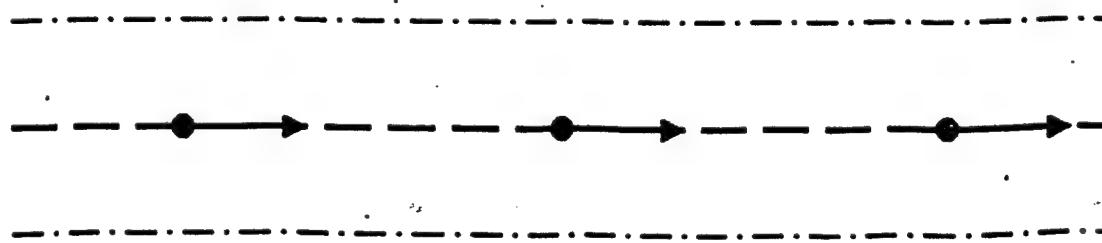


Fig.18. A staggered formation of a group of search vehicles.

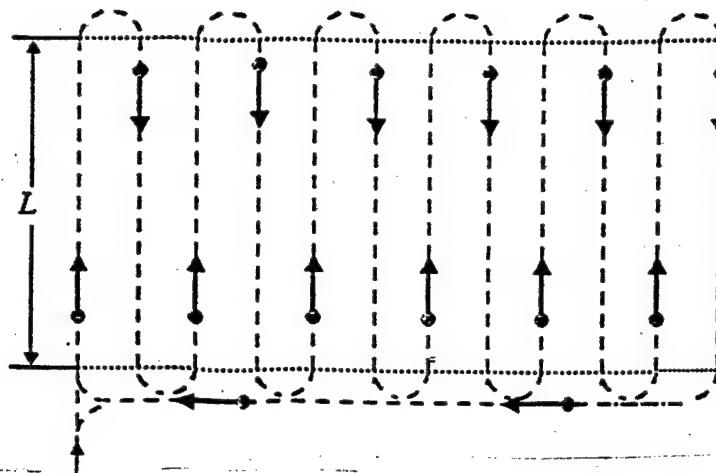


Fig.19. A staggered formation for the parallel strategy of searching.

The staggered formation decreases the period of the observation by a factor of  $1/n$ .

If the distance between the vehicles is less than  $a$  (Fig.7b), then the observation is continual.

Both methods ensure vehicle collision avoidance.

3. The area of observation is separated into regions and the group of vehicles is divided into subgroups. One subgroup serves one region.

If there are a lot of regions and vehicles, or insufficient number of their obviously all of regions of the cannot be covered by the given number of vehicles.

#### Searching on Area having Important Regions

The search area can have regions where there is a greater probability of a target being present or where it is thought that an important target located. How does one organize a search in this case? The simplest way is to decrease the period of observation in these regions. It suggests two methods:

- 1) To decrease the distance between the parallel trajectories in the sequence method (Fig.20) or additional closed-loop trajectories in the important areas (Fig.21).
- 2) The periods for the cases in Figs. 20-21 are depicted in Fig.22-23, respectively.

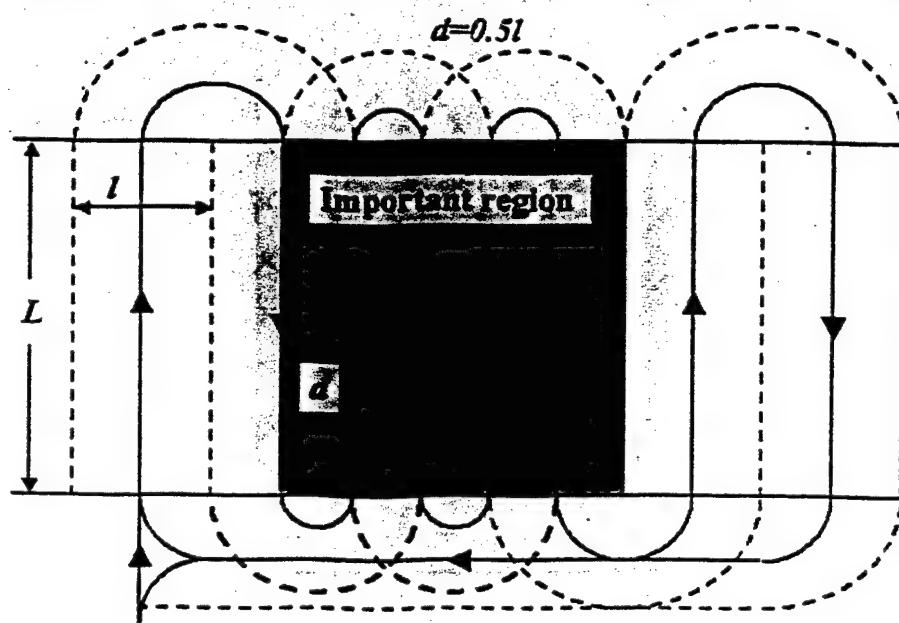


Fig.20. The first method decreasing the observation period in an important region.

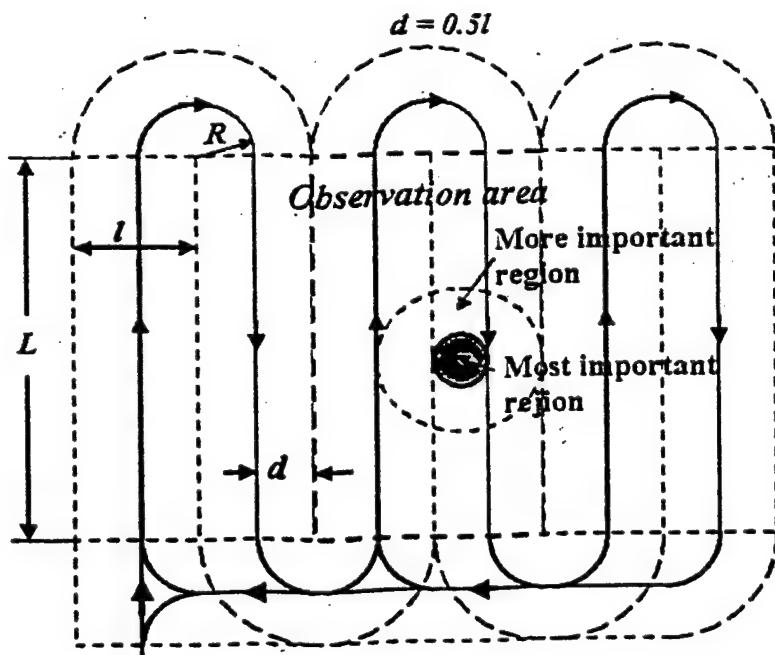


Fig.21 The second method decreasing the observation period in an important region:

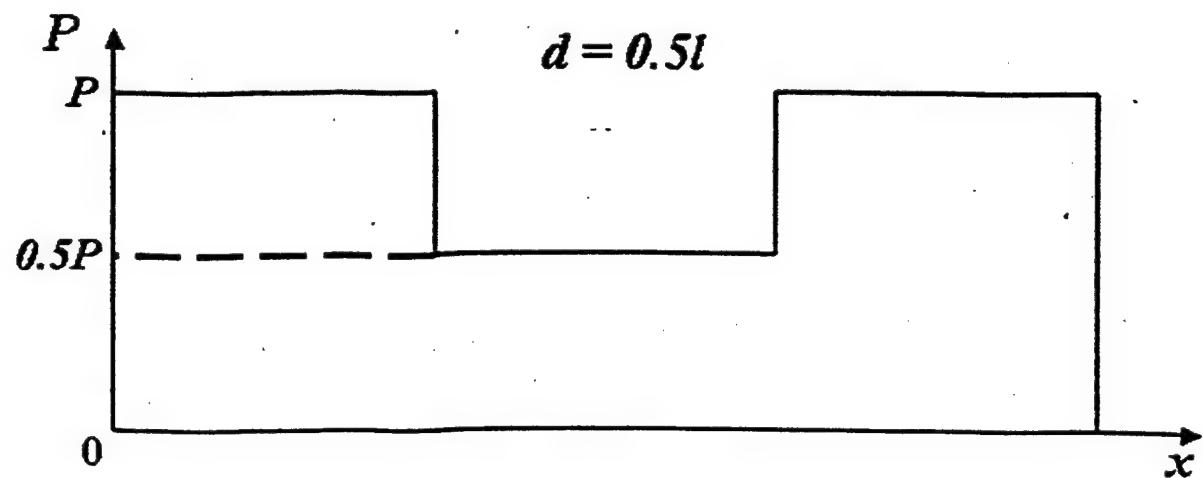


Fig.22. Period of the observation in an important rectangular area.

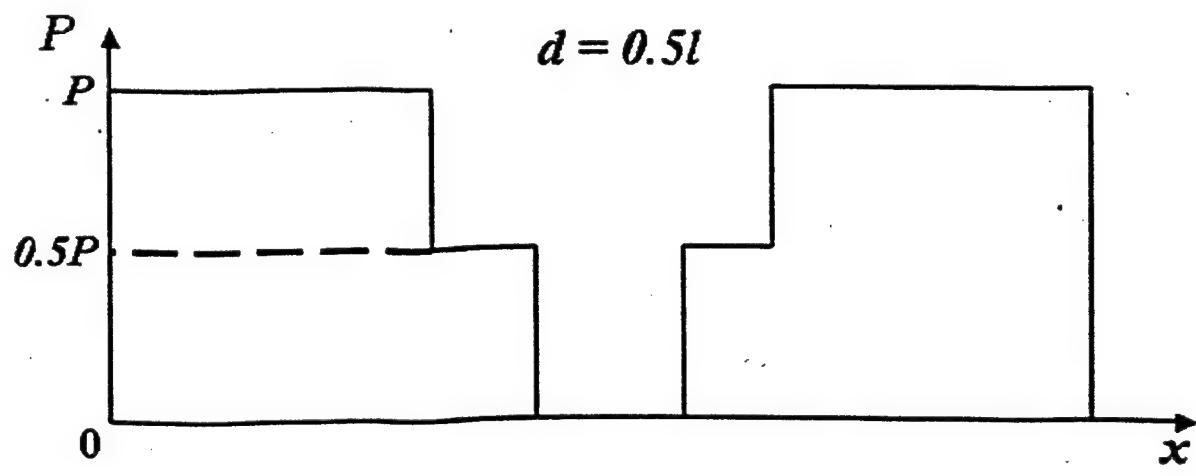


Fig.23. Period of the observation in the important round area. Cross-section is along of the diameter.

## 7. Selecting an Appropriate Attack Vehicle from a Group of Attack Vehicles

When the target is found, there is the problem of how to annihilate it as soon as possible or how to select the appropriate attack vehicle from the group of available attack vehicles. The attack vehicles may have different velocities and the nearest vehicle may be not well suited because his velocity could be in the opposite direction from the target. In Fig.20 a vehicle  $V1$  is very close to a target  $T$ , but it must make a turn before getting into a suitable position for attack. Any turn has a minimal radius  $R_{min}$  and the turn can require a lot of time. A vehicle  $V2$  located farther from the target  $T$  may be more suitable for attack than vehicle  $V1$  which is closest to the target.

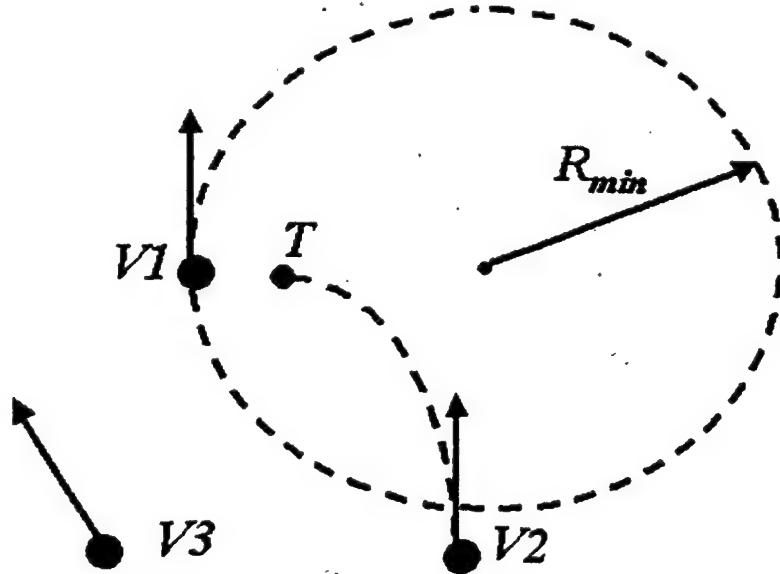


Fig.24. The choice of an attack vehicle for annihilation of a target.

We proceed to establish a formula which will aid in the initial selection.

### Case when the target is out of a reach circle.

- a) The target position is given by the distance from vehicle and the line of sight angle from the direction of vehicle velocity.

Referring to Fig.24, assume that an attack vehicle is located at point **A** with velocity **V** and a target is located at point **B**. The line of sight angle between the velocity vector **V** and the direction to the target **B** is  $\alpha$ . The distance between the attack vehicle and the target is **L**. Given that the minimal radius of turn is **R**, an admissible path **S** (**ADB**) can be found ( $L > 2R$ ):

$$S = R(\pi/2 + \alpha - \beta + \tan \beta - \cos \alpha / \cos \beta), \quad (29)$$

where  $\beta$  is the angle between the relative position vector **L** and a turn radius **OD** at the end of the turn. The angles  $\alpha$  and  $\beta$  are in radians.

The equation for  $\beta$  can then be derived as:

$$(1 - \cos \alpha / \sin \beta) / \cos \beta + \cos \alpha \cos \beta / \sin \beta + \sin \alpha - L/R = 0. \quad (30)$$

This equation has two solutions. The solution which yields the smaller angle  $\theta$  is

$$\theta = \pi/2 - \beta - \alpha.$$

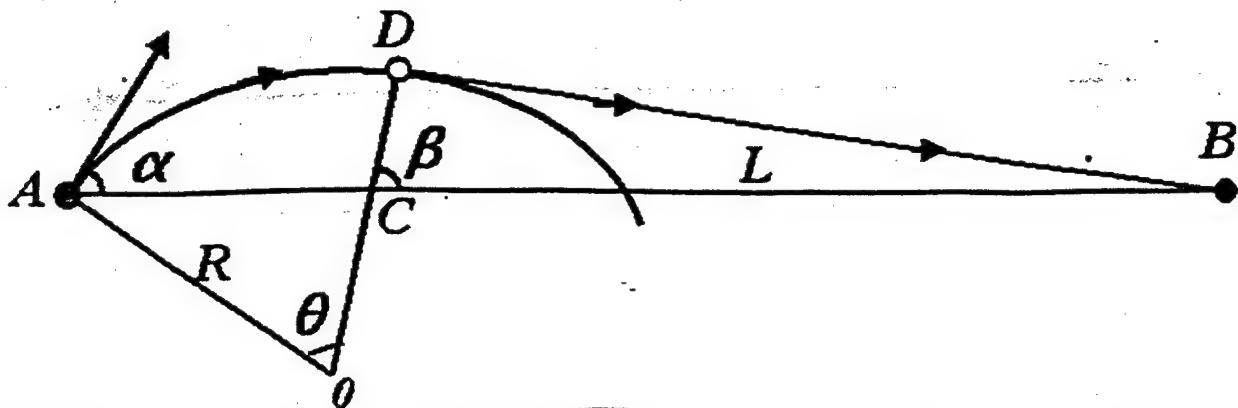


Fig.25. Finding the flight distance between a target and an attack vehicle using distance  $L$  from the vehicle to the target and the line of sight angle  $\alpha$ .

If the speed of the vehicle  $|V|$  is known and is assumed to be constant, then the time required to reach the target is

$$T = R (\pi/2 + \alpha - \beta + \tan \beta \cdot \cos \alpha \cdot \cos \beta) / V. \quad (31)$$

If the speed of the turn is  $V_1$  and the speed of straight flight is  $V_2$ , the intercept time is

$$T = R [(\pi/2 + \alpha - \beta) / V_1 + (\tan \beta - \cos \alpha \cdot \cos \beta) / V_2]. \quad (32)$$

Using equations (30) – (31), we can calculate the time required to reach the target by each attack vehicle and select the most suitable vehicle or vehicles. The graph

$$(S/R) = f(\alpha, L/R), \quad (33)$$

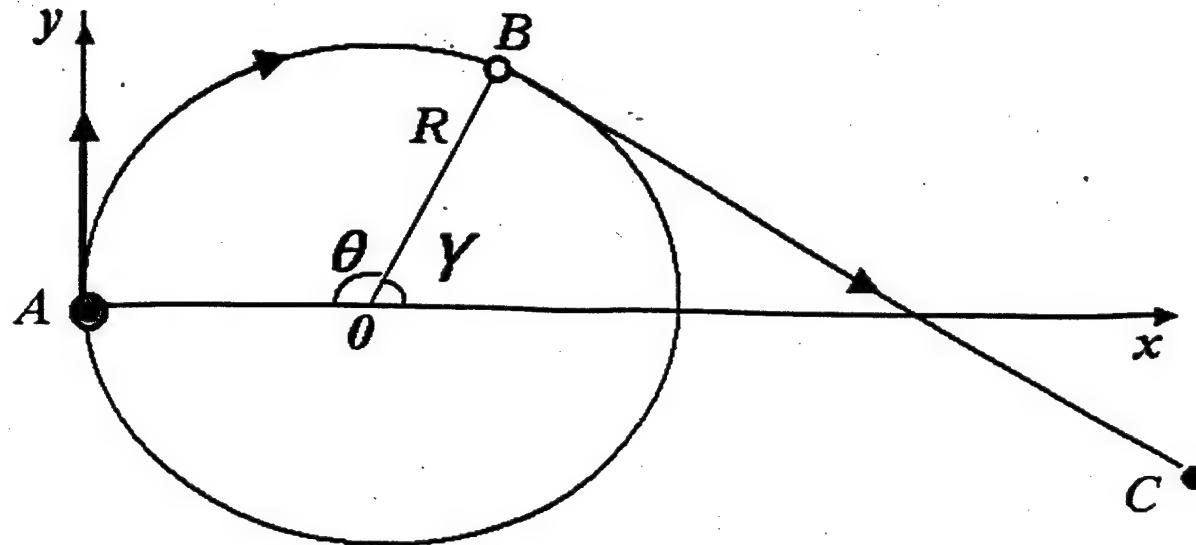
makes such a selection easy.

If a target is very important and we want to increase its probability of annihilation, the attack may involve two to three vehicles having a short time to reach it.

If  $\alpha$  is less 0.2, it is better to use the simpler equation  $T = L/V$  for estimation of the intercept time.

b) The target position is given in a rectangle coordinate system.

Let us locate the center of the coordinate system in the attack vehicle and let the  $y$ -axis be coincident with the vehicle velocity (Fig. 26).



Fog.26. Finding the flight distance between a target and the attack vehicle, using a rectangular system of coordinates.

In the  $x, y$  coordinates system:

- 1) the target is denoted by the sub index “ $c$ ”;
- 2) the center of the reach circle is denoted by the sub index ‘ $o$ ’.

Without loss of generality we assume the  $x$ -coordinates of the center of the reach circle and the target have the same signs.

From Fig.26, the equations for the coordinates  $x, y$  of the point  $B$  are

$$R^2 + (x_c - x)^2 + (y_c - y)^2 = (x_c - x_o)^2 + (y_c - y_o)^2, \quad (34)$$

$$(x - x_o)^2 + (y - y_o)^2 = R^2, \quad (35)$$

where  $R$  is the minimal turning radius of a vehicle in an horizontal plane.

These equations give two solutions (positions) of the point B. The solution which yields the smaller angle  $\theta$  is

$$\theta = \pi - \gamma, \quad (36)$$

where

$$\gamma = \arctan[y/(x - x_0)]. \quad (37)$$

The flight distance from the attack vehicle to the target is

$$S = R\theta + [(x_c - x_0)^2 + (y_c - y_0)^2 - R^2]^{0.5}, \quad (38)$$

where  $\theta$  is in radians.

If  $R \ll (x_c^2 + y_c^2)^{0.5}$ , the flight distance approximately equals

$$S \approx (x_c^2 + y_c^2)^{0.5} \quad (39)$$

**Case when the target is located within a reach circle.**

This case is more complex. We suggest the following solution (Fig.27).

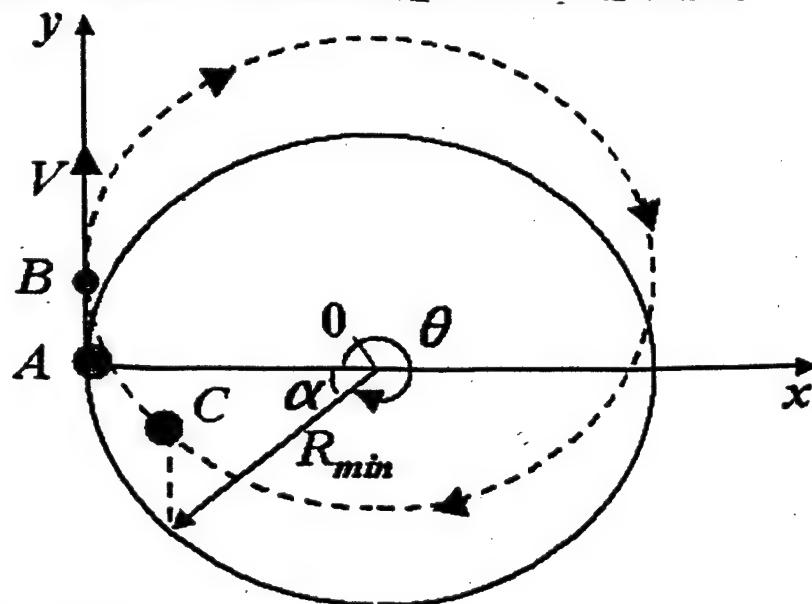


Fig.27. Case when the target is located inside of a reach circle of the attack vehicle.

Marks: A – attack vehicle, V – velocity vector, C – target.

**Strategy 1.** A plausible strategy of the attack vehicle is the following: the attack vehicle flies forward to point *B* until the target lies on the perimeter of the reach circle #2; the attack vehicle then begins turning flight.

In the coordinate system of Fig.27, the full flight distance equals

$$S = R\theta + DC, \quad (40)$$

where *DC* is distance between the points *D* and *C*; the angle  $\theta$  is in radians.

a) If  $x_c < 0$ , the coordinate of the point *D* is

$$y = - [R^2 - (x_c - x_o)^2]^{0.5} \quad (41)$$

The angle  $\theta$  is

$$\theta = 2\pi - \alpha \quad (42)$$

where  $\alpha$

$$\alpha = \arctan[y/(x_c - x_o)]. \quad (43)$$

The flight distance equals

$$S = R + (y_c - y) \quad (44)$$

b) If  $x_c > 0$ , the equation (42) is

$$\theta = \pi - \alpha \quad (45)$$

**Strategy 2.** For the same case above, the target can also be reached by making a left turn AGHC (Fig.28).

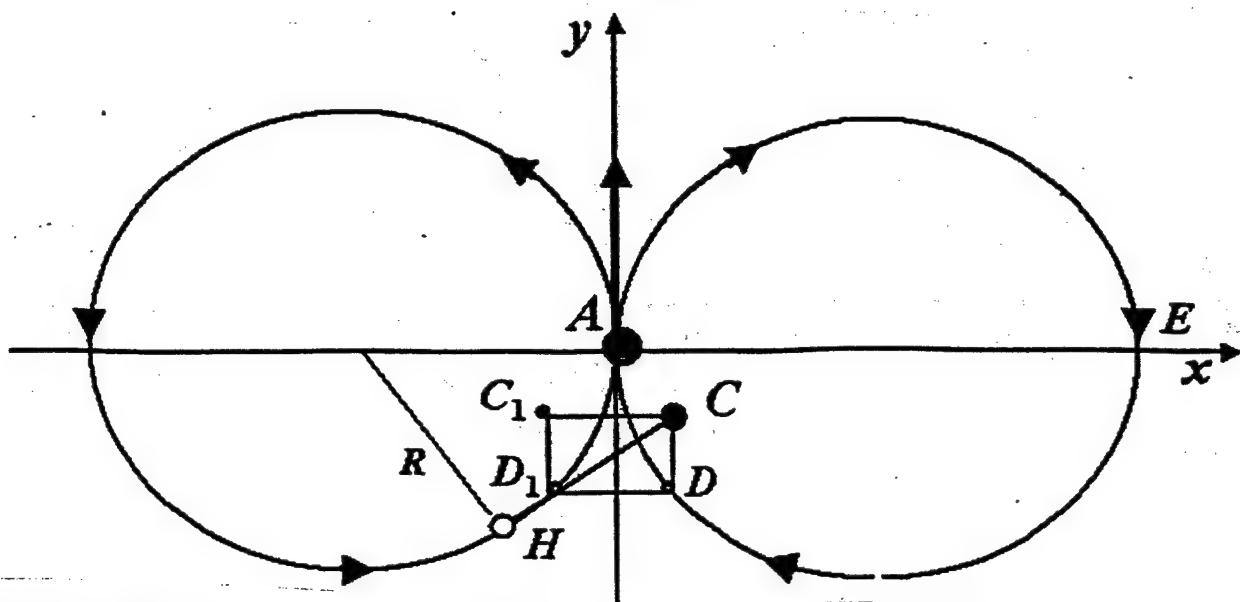


Fig.28. The reaching of a right target by right or left turns.

In this case the attack vehicle turns left turn until its velocity vector is in the direction of targets the vehicles that flies straight to the target.

We now show that this method is worse than the right turn if the vehicle speed is same in both cases.

Consider the Fig.28. The target is at the point  $C$ . Turning right, the vehicle has the equivalent path  $AEDC$ . Turning left the vehicle has the path  $AGHC$ . The segment  $HF > HD_1$ , and the segment  $FC > DC$ . This means the path  $AGHC$  is longer then the path  $AEDC$ .

## 8. General Rules of Behavior of the Attack Vehicle for Annihilation of Target

The previous consideration leads to a simple list of behavioral rules of the attack vehicle when a target is found.

**Case 1.** The target is located OUT of reach circle

- a) The vehicle turns into the half plane containing the target until its velocity vector is pointing at the target.
- b) The vehicle then flies straight to the target.

**Case 2.** The target is located **INSIDE** of the reach circle.

- a) The vehicle continues to flight ahead (it increases its distance from the target!) until the target lies on the perimeter of the reach circle.
- b) The vehicle then turns to intercept the target.

The turn must be into the half plane containing the target.

### Required Number of Search and Attack Vehicles

The needed number of search vehicles may be estimated in the following way.

assume that one vehicle can observe  $s$  units of a territory in one unit of time.

There are  $m$  connected areas  $S_i$  of observations with period  $T_i$  in each area. The number of vehicles needed for full observation all areas with periods  $T_i$  is

$$N = (1/s) \sum S_i / T_i \quad (i = 1, 2, \dots, m) \quad (46)$$

If number of vehicles  $N$  is given and it is less then the needed number  $N_n$ , an operator must increase the periods of the observation in  $N_n/N$  time.

### Some other problems, which can be solved in this field:

- 1) If observation areas have different locations and the locations are far from one another, we get a problem of optimal paths between them.
- 2) The problem of optimal target assignments.

## Summary

A number of problems associated with searching and attacking enemy targets have been presented. Solving these problems has led to the development of a) search strategies for both single and multiple air vehicles in pursuit of fixed and mobile targets, respectively, b) strategies for searching important areas within a region and for continually observing such areas, c) techniques for calculating the number of required air vehicles for various tasks, and d) a method for selecting the best vehicle from a group for the purpose of attacking a target.

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